Design and Evaluation of a MEMS-Based Stirling Microcooler

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ABSTRACT

A new Stirling micro-refrigeration system composed of arrays of silicon MEMS cooling elements has been designed and evaluated. The cooling elements are to be fabricated in a stacked array on a silicon wafer. A regenerator is placed between the compression (hot side) and expansion (cold side) diaphragms, which are driven electrostatically. Air at a pressure of 2 bar is the working fluid and is sealed in the system. Under operating conditions, the hot and cold diaphragms oscillate sinusoidally and out of phase such that heat is extracted to the expansion space and released from the compression space. Parametric study of the design shows the effects of phase lag between the hot space and cold space, swept volume ratio between the hot space and cold space, and dead volume ratio on the cooling power. Losses due to regenerator non-idealities are estimated and the effects of the operating frequency and the regenerator porosity on the cooler performance are explored. The optimal porosity for the best system COP is identified.

Keywords: Stirling microcooler, Regenerator, COP

NOMENCLATURE

\begin{itemize}
\item $A$ Convective heat transfer area in the chamber, m\textsuperscript{2}
\item $A_w$ Wetted area of the pillars, m\textsuperscript{2}
\item $COP$ Coefficient of performance
\item $F$ Forchheimer coefficient
\end{itemize}
\( K \)  
Permeability, m\(^4\)

\( NTU \)  
Number of transfer units

\( Nu \)  
Nusselt number

\( Pr \)  
Prandtl number

\( Q \)  
Heat extraction per cycle, J

\( Re \)  
Reynolds number

\( T \)  
Temperature, K

\( U \)  
Average velocity, m/s

\( V \)  
Volume, m\(^3\)

\( W \)  
Work, W

\( c_p \)  
Specific heat, J/(kg·K)

\( d \)  
Pillar diameter, m

\( e \)  
Regenerator effectiveness

\( f \)  
Operating frequency, Hz

\( h \)  
Convection coefficient, W/(m\(^2\)·K)

\( k \)  
Thermal conductivity, W/(m·K)

\( \dot{m} \)  
Gas mass flow rate, kg/s

\( p \)  
Pressure, Pa

\( q \)  
Rate of heat transfer, W

\( q_{cd} \)  
Heat conduction loss across regenerator walls, W

\( \alpha \)  
Lead phase angle of the cold side to the hot side

\( \Delta p \)  
Pressure drop through the regenerator, Pa

\( \delta_t \)  
Thermal penetration depth, m
1. INTRODUCTION

Thermoelectric coolers are commonly used for chip- and board-level electronics cooling [1]. Thermoelectric energy conversion has low efficiency, however, resulting in significant power requirements and waste heat generation. Furthermore, significant challenges exist in the design and fabrication of thermoelectric materials [2], leading us to explore an alternative option: the Stirling microcooler. Micro-scale devices operating on the Stirling cycle are an attractive potential alternative based on the high efficiencies realized for macro-scale Stirling machines [3]. Attempts to miniaturize Stirling coolers for application in electronics cooling have been scale-
limited by the use of traditional components (e.g., pistons, linkages, and pressure vessels) [4, 5].
As a result, Stirling coolers have been impractical for most electronic packaging applications. The state-of-the-art in small, but conventionally manufactured, Stirling coolers is exemplified by an on-board system measuring $13 \times 25 \times 6 \text{ cm}^3$ in the palm-size Titanium infrared imagers made by FLIR [6]. Previous efforts to design a micro-scale Stirling cooler addressed frictional losses and leakage concerns by replacing conventional pistons and the associated linkages with electrostatically-driven diaphragms [7-9]. The Moran concept [8, 9] used 1-2 cm diameter silicon diaphragms with vertical electrostatic comb drives to move the working gas through a layered metal/polymer regenerator, as illustrated in Fig. 1(a). The substantial heat loss from the hot side to the cold side is challenging to minimize.

In this paper, we report on the design of a new micro-scale Stirling cooler system, which includes two diaphragm actuators and a regenerator that separates the hot and cold chambers. Each element is designed such that the gas flow direction is within the plane of the silicon wafer, as shown in Fig. 1(b). This in-plane micro-scale implementation offers efficient thermal isolation between the hot and cold sides by enabling a longer length regenerator than may be practical in a stacked assembly meant to support gas flow perpendicular to the wafer, as shown in Fig. 1(a). To achieve a larger heat lift, an arbitrary number of cooler elements in our design can be assembled in parallel. The rest of the paper is organized as follows. In Section 2, the detailed design concept of the new Stirling microcooler is discussed. A system-level thermodynamic analysis of the system is then presented in Section 3 and the effects of non-idealities on the cooler performance are quantified in Section 4 using detailed fluid dynamics and heat transfer analyses.

2. DESIGN CONCEPT

2.1 Overall Concept
The Stirling microcooler elements are each 5 mm-long, 2.5 mm-wide, have a thickness of 150 μm, and are fabricated on a silicon wafer, as illustrated in Fig. 2(a). The design minimizes conduction heat losses across a 0.5 mm-long regenerator by distancing the compressor and expander assemblies with a low thermal conductivity passage. The working fluid (e.g., air) is pressurized at 2 bar. The compression and expansion processes are driven by 2.25 mm diameter circular diaphragms that are actuated electrostatically. The assembled structure is comprised of a pair of identical planar elements positioned face-to-face, as shown in Fig. 2(b), and has five parts: the diaphragm layer in the middle, the top and bottom chamber substrates, and two sealing layers. Each of these parts is attached with activated bonding of polydimethylsilicone (PDMS) to either silicon or silicon oxide layers. The cooling elements are designed to be arrayed along their width and thickness to create larger-area surfaces with cooling capacity that scales with their cross-section area. For example, the $2 \times 2$ cm$^2$ cooling area shown in Fig. 3 is made by first arraying eight elements along their width and then stacking 114 of these $1 \times 8$ arrays. This active cooling module is reduced in volume by three orders of magnitude compared to a standard small-scale (2000 cm$^3$) Stirling cooler.

2.2 Hot and Cold Chambers

Under operating conditions, the hot and cold diaphragms oscillate sinusoidally and out of phase such that heat is extracted from the source (which is at a lower temperature than the sink) to the expansion space and released to the sink (typically the surroundings) from the compression space. The diaphragms are actuated electrostatically to drive the working fluid, which transfers heat between the two chambers. In this design, the bulk silicon substrate on which the device is grown is etched with zipper-shaped chambers under the diaphragms. The silicon substrate enables efficient heat transfer between the gas and heat source/sink and the zipper shape of the
substrate reduces the pull-in voltage required to actuate the diaphragms. The addition of silicon pillars at the outlet of each chamber also enhances heat transport from the gas, as shown in Fig. 2(a).

2.3 Regenerator

The flow channels of the 0.5 mm-long regenerator are positioned along the wafer plane connecting the compression and expansion chambers. The top and bottom walls of the regenerator are made from thin-film silicon oxide and coated with PDMS. Both materials’ thermal conductivities are about one hundred times less than that of silicon. The sidewalls are made from SU8 epoxy, which also has low thermal conductivity. As a result, the conductive heat leakage from the hot section to the cold section is small (see Section 4.2.2). A series of vertical silicon pillars (diameter 20 µm) are fabricated in the regenerator and serve as the thermal capacitor, which enables efficient heat transfer to/from the gas as it passes through.

2.4 High-Frequency Operation

The cooling capacity of a Stirling cooler is directly related to its operating frequency. Small thermal penetration depths make high frequency-regenerators impractical in large coolers. In microcoolers, however, high-frequency operation is possible, as the length scales of the components of the regenerator can be made comparable to the thermal penetration depth, \( \delta_t \), which is given by [10],

\[
\delta_t = \sqrt{\frac{k}{\pi f \rho c_p}}.
\]  

(1)

where \( k \) is the thermal conductivity of the silicon, \( f \) is the operating frequency, \( \rho \) is the silicon density and \( c_p \) is the specific heat of the silicon. At an operating frequency of 2 kHz, the thermal penetration depth in silicon is 122 µm at a temperature of 20°C. Therefore, the diameters of the
pillars in the regenerator should thus not exceed 122 µm. Much smaller feature sizes (down to about 20 µm) are accessible though microfabrication.

3. THERMODYNAMIC ANALYSIS OF THE SYSTEM

3.1 Stirling Cycle

The basic Stirling cooling concept employs pumps (either pistons or diaphragms) to drive a working fluid between a cold region and a hot region across a regenerative heat exchanger (i.e., the regenerator). The operation of an ideal Stirling refrigeration cycle is illustrated in Fig. 4. Also shown are the corresponding pressure-volume and temperature-entropy diagrams. The ideal cycle starts with an isothermal compression of the working gas from states 1 to 2 that increases the pressure and decreases the gas volume, thereby sending heat to the surrounding chamber. The gas is then cooled in a constant volume process as it is forced through the regenerator into the expansion space (states 2 to 3) while dumping the heat to the regenerator. From states 3 to 4, the working gas is expanded in an isothermal process that decreases the pressure and increases the gas volume, thereby extracting heat from the surrounding chamber. The cycle returns to its original state (states 4 to 1) with the gas absorbing heat from the regenerator. During steady-state operation, this cycle produces a cold region by extracting heat in the expansion space for cooling and a hot region by releasing heat in the compression space. In reality, the Stirling refrigerator will not operate as an ideal cycle, but rather in a sinusoidal-like motion with a phase lag between the cold and hot sides. The variations of the cold side, hot side, and total system volumes of this cycle are shown in Fig. 5.

3.2 Thermodynamic Analysis of the Stirling Cycle

In a cooler driven by the Stirling cycle, the dead volume ratio and the swept volume ratio between the two chambers play an important role in determining the cooling capacity [11]. The
dead volume, $V_D$, includes the regenerator volume, $V_r$, and the non-swept volume between the regenerator and the chambers, $V_{ns}$. These volumes are shown in Fig. 6. $V_C$ and $V_H$, which are also shown in Fig. 6, are the swept volume of the cold and hot chambers. $\kappa = V_H/V_C$, is the swept volume ratio, and $\chi = V_D/V_C$ is the dead volume ratio. In order to understand the effects of these volume-related parameters on the Stirling cooler, its performance can be approximated with a first-order analysis without considering any losses in the system. Several assumptions are made during this procedure. We assume that the regenerator is perfect, such that at the outlet of the regenerator, the gas temperature is the same as the chamber temperature (i.e., the regenerator effectiveness is unity). The pressure, $p$, in the system at any point in time is uniform. The gas temperature in each chamber is uniform and constant, and the temperature profile in the dead space is linear. If the phase lag between the cold side and the hot side is $\alpha$, then the volume variation of the cold space as a function of the phase angle $\varphi$, is

$$V_c = \frac{1}{2}V_C(1 + \cos \varphi),$$

and the volume variation of the hot space is

$$V_h = \frac{1}{2}V_H[1 + \cos(\varphi - \alpha)].$$

The total mass of the working fluid in the system, $m$, is constant, so that, using the ideal gas law,

$$\frac{mR}{p} = \frac{V_c}{T_{g,C}} + \frac{V_h}{T_{g,H}} + \frac{V_D}{T_{g,D}},$$

where $T_{g,C}$ is the average cold gas temperature and $T_{g,H}$ is the average hot gas temperature. The mean temperature in the dead space, $T_{g,D}$, is

$$T_{g,D} = \frac{1}{2}(T_{g,H} + T_{g,C}).$$

The expansion and compression processes take place isothermally, so that the heat released or absorbed in each chamber is equal to the work, such that
\[ Q = \int p \, dV. \quad (6) \]

Based on Eqs. (2)-(6), the variation of the system pressure as a function of the phase angle \( \varphi \) is

\[ p = \frac{p_{\text{max}}(1-\delta)}{1+\delta \cos(\varphi-\theta)}. \quad (7) \]

Using Eqs. (6) and (7), the heat extracted by the gas in the cold chamber per cycle is then

\[ Q_C = \frac{\pi \delta \sin \theta \sqrt{1-\delta}}{(1+\kappa \sqrt{1+\delta})(1+\sqrt{1-\delta^2})} p_{\text{max}} (V_C + V_H). \quad (8) \]

The heat released by in the hot chamber per cycle is

\[ Q_H = \tau Q_C. \quad (9) \]

In Eqs. (7)-(9), \( p_{\text{max}} \) is the maximum pressure, \( \delta \) is an intermediate variable, 

\[ \frac{\sqrt{1+\kappa^2+2\kappa \cos \alpha}}{\tau+\kappa+2S}, \]

where \( \tau = T_{g,H}/T_{g,C} \) is the gas temperature ratio, and \( S = 2\tau \chi/(1+\tau) \) is the reduced dead volume. The variable \( \theta = \tan^{-1} \left( \frac{\kappa \sin \alpha}{\tau+\kappa \cos \alpha} \right) \), can be interpreted as the lead phase angle of the volume variation in the cold side to the pressure variation in the element. If the initial pressure in the system is \( p_0 \), then the maximum pressure in the system is obtained, based on Eq. (7), to be

\[ p_{\text{max}} = p_0 \frac{1+\kappa+2\chi}{1+\kappa+2\chi-\sin \alpha} \frac{1-\delta \sin(\alpha-\theta)}{1-\delta}. \quad (10) \]

From Eqs. (8) and (10), the dimensionless heat extraction \( Q^*_C \) is

\[ Q^*_C = \frac{Q_C}{p_0 (V_C + V_H)} = \frac{\pi \delta \sin \theta [1-\delta \sin(\alpha-\theta)]}{(1+\kappa \sqrt{1+\delta})(1+\sqrt{1-\delta^2})} \frac{1+\kappa+2\chi}{1+\kappa+2\chi-\sin \alpha}. \quad (11) \]

For a 25 K temperature lift, \( T_{g,H} = 313.15 \text{ K} \) and \( T_{g,C} = 288.15 \text{ K} \), the dimensionless heat extraction per cycle is a function of the swept volume ratio \( \kappa \), the phase lag between the cold side and the hot side \( \alpha \), and the dead volume ratio \( \chi \). For the dead volume, we fix the regenerator volume and study the effect of varying the non-swept volume. Note that the non-swept volume ratio is \( \chi_{V_{ns}} = V_{ns}/V_C \). In Figs. 7 and 8, the dimensionless heat extraction per cycle based on variation of \( \kappa, \alpha \) and \( \chi_{V_{ns}} \) is plotted. In Fig. 7, the effect of the swept volume ratio \( \kappa \) on the
dimensionless heat extraction at different non-swept volume ratios when the lead phase angle of the cold side to the hot side is 90° is shown. The curves show that an optimum value of κ can be found at which the dimensionless heat extraction is maximized. This optimum value of κ is always unity for this concept, which corresponds to the cold side and hot side having the same swept volume. The same swept volumes maximize the heat extracted or released in the cold or hot chamber. The effect of the non-swept volume ratio is clear from Fig. 7: with an increase of \( \chi_{v_{ns}} \), the dimensionless heat extraction decreases. This result indicates that the non-swept volume should, as expected, be as small as possible. In Fig. 8, the effect of lead phase angle \( \alpha \) on the dimensionless heat extraction at different swept volume ratios when the non-swept volume ratio is 0.4 is plotted. From this figure, we can see that the dimensionless heat extraction varies by only \( \pm 10\% \) when the lead phase angle is in the range of 60° to 110°.

4. SYSTEM-LEVEL LOSS EVALUATION

4.1 Coefficient of Performance

In Section 3, we assumed that the system was perfect and did not consider any losses. In the real system, however, non-idealities will be present and their effects are now considered. Note that in Section 3 we use \( Q \) to represent the heat extraction per cycle. Here we use \( q \) as the rate of the heat transfer. We start with the coefficient of performance (COP) of the cooler, which is the ratio of the heat removal to the input work,

\[
COP = \frac{q}{W}
\]

The Carnot coefficient of performance \( COP_C \), which represents the maximum theoretical efficiency possible between constant temperature heat source \( T_C \) and heat sink \( T_H \), is [11]

\[
COP_C = \frac{T_C}{T_H - T_C}
\]

For \( T_H =313.15 \) K and \( T_C =288.15 \) K, \( COP_C \) is 11.5. An estimate of the actual coefficient of
performance requires knowledge of the real work input and the heat loss. In our design, the losses come from: (i) Convective heat transfer resistance between the silicon substrate and the gas in the chamber, (ii) Heat conduction along the regenerator walls, (iii) Insufficient heat transfer in the regenerator, and (iv) Pressure drop through the regenerator.

4.2 Energy Balance

To estimate the actual cooling power $q$ of the system, a simplified model is built and an energy balance is made. As shown in Fig. 9, the regenerator is not perfect, so the gas in the cold side not only absorbs heat from the heat source, but also gets some heat from the gas coming out of the regenerator, $q_r$. Along the regenerator walls, there is heat conduction loss from the hot side to the cold side, $q_{cd}$. Combining these losses, we obtain an energy balance for the cold chamber

$$ q_C = q + q_{cd} + q_r, \quad (14) $$

where $q_C$ is total heat absorbed by the cold gas when the cold gas space expands. It can be calculated from Eq. (8). Similarly, for the hot chamber,

$$ q_H = q_h + q_{cd} + q_r, \quad (15) $$

where $q_h$ is the actual heat rate coming out of the hot chamber and $q_H$ is total heat rate released by the hot gas when the hot gas space is compressed. It can be calculated from Eq. (9).

4.2.1 Heat Transfer in the Chamber

The heat source temperature is $T_C$, the heat sink temperature is $T_H$, the convection coefficient between the cold/hot chamber and the heat source/sink is $h$, and the surface area between them is $A$. Due to the high thermal conductivity of silicon, we assume the substrate in each chamber is isothermal. Then, the actual cooling power for the system is

$$ q = hA(T_C - T_{g,c}). \quad (16) $$
In the hot chamber, the heat coming out of the hot chamber is

\[ q_h = hA(T_{g,H} - T_H). \]  

(17)

\( T_{g,C} \) and \( T_{g,H} \) are the gas temperature in the cold and hot chambers. The convection coefficient in the chamber is obtained from the COMSOL multiphysics simulation of the Stirling microcooler system [12]. The Nusselt number used for the calculation is \( Nu = 0.316Re^{0.31} \). In the range of the present operating frequency (100 Hz ~ 1000 Hz), the Reynolds number range is about 10 ~ 100. As the chamber structure is complex, here the hydraulic diameter is used for the Nusselt number and Reynolds number.

4.2.2 **Regenerator Heat Conduction**

The top and bottom walls of the regenerator are made from thin-film silicon oxide and coated with PDMS. The sidewalls are made from SU8. All these materials have low thermal conductivity: \( k_{SiO_2} = 1.4 \) W/(m·K), \( k_{SU8} = 0.3 \) W/(m·K) and \( k_{PDMS} = 0.2 \) W/(m·K) at a temperature of 20°C. The thicknesses of thin-film silicon oxide, SU8, and PDMS are 2 µm, 50 µm and 12 µm. As a result, the heat conduction loss \( q_{cd} \) is

\[ q_{cd} = \sum_{i=1}^{3} k_i A_{ci} \frac{T_{g,H} - T_{g,C}}{L_r}, \]  

(18)

where \( i = 1, 2, 3 \) represent the three different materials, \( A_c \) is the cross-sectional area, which is the thickness multiplied by the width of the regenerator wall, and \( L_r \) is the regenerator length.

4.2.3 **Regenerator Ineffectiveness**

As the regenerator is not perfect, the temperature of the gas leaving the regenerator will not be the chamber temperature. Thus, the heat required to raise the gas temperature is

\[ q_r = (1 - e)\dot{m}c_p(T_{g,H} - T_{g,C}). \]  

(19)

where \( \dot{m} \) is the average mass flow rate in the cooler and \( e \) is the regenerator effectiveness. As a first-order estimation, the NTU method is used here to estimate the regenerator effectiveness.
NTU, which is the number of transfer units, is a dimensionless parameter that is widely used for heat exchanger analysis and it is defined as

\[ NTU = \frac{h_r A_w}{m c_p}, \]

where \( h_r \) is the convection coefficient in the regenerator, and \( A_w \) is the wetted area in the regenerator. From an energy balance (i.e., setting the change of the gas enthalpy equal to the heat transfer between the gas and the regenerator), we obtain the regenerator effectiveness to be [14]

\[ e = \frac{NTU}{1 + NTU}. \]

The convection coefficient of the regenerator pillars with the gas can be estimated according to Zukauskas’s correlation which is widely recognized by the researchers [15],

\[ Nu_d = 0.9 Re_d^{0.4} Pr^{0.36}, (10 < Re_d < 100). \]

The Nusselt number and Reynolds number are defined as \( Nu_d = \frac{h_{rd}}{k} \) and \( Re_d = \frac{\rho u_{max} d}{\mu} \), where \( u_{max} \) is the maximum velocity through the regenerator, \( \mu \) is the dynamics viscosity, and \( Pr \) is the Prandtl number. \( d \) is the pillar diameter, which is 20 \( \mu m \) in our design.

Predictions of the regenerator effectiveness from the NTU method and from COMSOL multiphysics simulations [12] for regenerator porosities of 0.798, 0.864, and 0.916 are presented in Table 1. In the present regenerator, the Knudsen number is less than 0.01. The continuum theory is used for the COMSOL simulations. The maximum difference between the analytical model and the detailed numerical calculations is less than 5%, justifying the NTU method as a simple and useful tool for predicting the regenerator effectiveness. From the result, we can see that the regenerator effectiveness decreases with an increase of the operating frequency and increases with a decrease of the porosity. At a high frequency, the heat transfer between the solid and the gas is limited due to the small interaction time. When the porosity is large, the solid-gas
interface area is small and the regenerator effectiveness is poor.

4.2.4  Regenerator Pressure Drop

The work loss due to the pressure drop across the regenerator, $W_{loss}$, is modeled as

\[ W_{loss} = \Delta p f(V_c + V_H), \]

(23)

where $\Delta p$ is the pressure drop through the regenerator. To predict the pressure drop in the regenerator (which is a microchannel containing a pillar matrix), a porous medium model is used in a COMSOL multiphysics simulation [12]. The relationship between the pressure drop and the gas velocity is then correlated by the permeability $K$ and Forchheimer coefficient $F$[16-18]. The one-dimensional Darcy-Forchheimer equation is

\[ \frac{\Delta p}{L} = \left( \frac{d^2}{K} + F \ Re_d \right) \frac{\mu U}{d \varepsilon}, \]

(24)

where $U$ is the average velocity. Lee proposed a correlation of the Darcy drag $\frac{d^2}{K}$ and the Forchheimer coefficient $F$ for a bank of the cylinders, given by [19]

\[ \frac{d^2}{K} = \frac{31 (1-\varepsilon)^{1.3}}{\varepsilon^3 (\varepsilon - 0.2146)}, \]

(25)

\[ F = \frac{(1-\varepsilon)^{1.4}}{\varepsilon^3 (\varepsilon - 0.2146)} \sum_{n=1}^{3} \sum_{m=1}^{3} a_{mn} \varepsilon^{m-1} Re_d^{n-1}, \]

(26)

where $a_{mn} = \begin{bmatrix} 4.825 & -0.166 & 0.001777 \\ -17.754 & 0.5893 & -0.00616 \\ 15.911 & -0.4736 & 0.004836 \end{bmatrix}$, and $\varepsilon$ is the regenerator porosity.

The predictions of the pressure drop from Lee’s correlation and from COMSOL multiphysics simulation [12] are shown in Table 2. The maximum difference is less than 15%, indicating that the correlation of Lee provides a good estimate to the full system behavior. The pressure drop increases with increasing frequency and decreasing porosity. The higher frequency makes the velocity higher and the pressure drop also increases. Decreasing porosity also
increases the velocity and increase the contact area at the same time, leading to a higher pressure drop.

4.3 System Evaluation

Using the results from Sections 4.2, the cold gas and the hot gas temperature can be obtained by an iterative calculation by combining Eqs. (8), (9) and (14)-(22). If the gas temperature is known, then the actual cooling power of the cooler is obtained from Eq.(16). The work loss is calculated from Eqs.(23)-(26). Then, the actual COP of the system can be estimated using Eq. (12). Here, our goal is to examine the effects of the regenerator porosity and the operating frequency on the system performance. The porosity of the non-swept space between the regenerator and the chamber space is fixed at 0.89. Based on the parametric study described in Section 3, the swept volume ratio is unity and the phase lag of the volume variations between the cold side and the hot side is set to 90°.

The system COP as a function of the regenerator porosity at different operating frequencies is plotted in Fig. 10. For a given frequency, an optimal porosity exists that maximizes the COP. This value is different at different frequencies. Generally, the porosity range of 0.8~0.9 gives the best efficiency for the microcooler. For the same porosity, the COP will always decrease with increasing frequency, as the heat transfer efficiency of the regenerator is reduced and the flow resistance through the regenerator increases. Both of these factors hurt the COP. When the operating frequency is 200 Hz, the COP of the system could be 5.4; however, the best COP is only 2.12 when the frequency is 1000 Hz. If the frequency is less than 600 Hz, then the best COP could be higher than 3, which is near to 30% percent of the Carnot COP.

Although the COP is low at a high frequency, with an increase of the frequency, the cooling capacity will increase. The system cooling capacity as a function of operating frequency
at different regenerator porosities is shown in Fig. 11. The cooling capacity increases when the porosity decreases. One reason for this trend is that the dead volume ratio decreases when the regenerator porosity decreases. Another reason is due to a decrease in the regenerator heat loss, as the regenerator effectiveness increases with decreasing the porosity.

5. CONCLUSIONS

In this paper, a new Stirling micro-scale cooler element has been designed and evaluated. The in-plane design, described in Section 2, separates the hot and cold chambers by a regenerator and the gas flow direction is within the plane of the wafer, which provides an excellent thermal isolation.

A thermodynamic analysis of the system and parametric studies of geometrical parameters were described in Section 3. As shown in Figs. 7 and 8, to improve the cooling power, the dead volume should be minimized, the swept volume ratio for the two chambers should be unity, and the phase lag of the volume variations between the cold side and the hot side should be 90°.

In Section 4, an analytic method was developed and applied to estimate the effects of system non-idealities (specifically related to the regenerator) on the coefficient of performance. The effects of the regenerator porosity and the operating frequency on the cooler performance were then studied. As shown in Figs. 10 and 11, increasing the frequency reduces the COP of the system while increases its cooling capacity, and the optimal porosity for the COP is 0.8~0.9.

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REFERENCES


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Table Captions

1. Table 1. Comparison of the regenerator effectiveness at different operating frequencies from an analytical model [Eq. (21)] and COMSOL multiphysics simulations [12]. When the porosity is 0.798, 0.864, or 0.916, the maximum difference between the two predictions is less than 5%, giving confidence to the use of the much simpler analytical model.

2. Table 2. Comparison of the pressure drop through the regenerator at different operating frequencies calculated from an analytical model [Eqs. (24)-(26)] and COMSOL multiphysics simulations [12]. When the porosity is 0.798, 0.864, or 0.916, the maximum difference does not exceed 15%, giving confidence to the use of the much simpler analytical model.
Table 1. Comparison of the regenerator effectiveness at different operating frequencies from an analytical model [Eq. (21)] and COMSOL multiphysics simulations [12]. When the porosity is 0.798, 0.864, or 0.916, the maximum difference between the two predictions is less than 5%, giving confidence to the use of the much simpler analytical model.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>COMSOL Simulation [12]</th>
<th>NTU [Eq. (21)]</th>
<th>Error (%)</th>
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<td>0.600</td>
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Table 2. Comparison of the pressure drop through the regenerator at different operating frequencies calculated from an analytical model [Eqs. (24)-(26)] and COMSOL multiphysics simulations [12]. When the porosity is 0.798, 0.864, or 0.916, the maximum difference does not exceed 15%, giving confidence to the use of the much simpler analytical model.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>COMSOL Simulation (Pa)[12]</th>
<th>Eqs. (24)-(26) (Pa)</th>
<th>Error (%)</th>
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Figure Captions

1. Figure 1. Stirling microcooler alternatives in cross section: (a) The Moran concept [8, 9] uses silicon diaphragms with vertical electrostatic comb drives to move the working gas through a regenerator vertically (i.e., perpendicular to the plane of the silicon wafer). (b) The current in-plane micro-scale implementation positions the regenerator flow channel parallel to the wafer plane connecting the compression and expansion chambers, allowing for thermal isolation.

2. Figure 2. Solid-model view of the Stirling microcooler elements. (a) A single element is 5 mm-long, 2.5 mm-wide, has a thickness of 150 μm, and is fabricated on a silicon wafer. (b) The assembled structure has five parts: the diaphragm layer in the middle, the top and bottom chamber substrates, and two sealing PDMS layers.

3. Figure 3. Vision of the arrayed Stirling microcooler. A 2 × 2 cm² cooling area is made by first arraying eight elements along their width and then stacking 114 of these 1 × 8 arrays.

4. Figure 4. The ideal Stirling refrigeration cycle includes four processes: isothermal compression (1-2), regenerative cooling (2-3), isothermal expansion (3-4) and regenerative heating (4-1).

5. Figure 5. Sinusoidal volume variation of the cold side, hot side, and the total system volume in the Stirling cycle. The numerical labels are similar to those in Fig. 4.

6. Figure 6. Geometric parameters for a microcooler element. The dead volume, \( V_D \), includes the regenerator volume, \( V_r \), and the non-swept volume between the regenerator and the chambers, \( V_{ns} \). \( V_C \) and \( V_H \) are the swept volume in the cold and hot chambers.

7. Figure 7. Dimensionless heat extraction as a function of swept volume ratio for different non-swept volume ratios. The lead phase angle of the cold side to the hot side is 90°.
8. Figure 8. Dimensionless heat extraction as a function of lead phase angle for different swept volume ratios. The non-swept volume ratio $\chi_{n_s}$ is 0.4.

9. Figure 9. System sketch of the single Stirling microcooler element and the energy balance in each chamber. $T_{g,C}$ and $T_{g,H}$ are the gas temperature in the cold and hot chambers. The heat source temperature is $T_C$ and the heat sink temperature is $T_H$.

10. Figure 10. System $COP$ as a function of regenerator porosity at different operating frequencies. The swept volume ratio is unity for the two chambers and the phase lag of the volume variations between the cold and hot sides is 90°.

11. Figure 11. System cooling capacity as a function of the operating frequency at different porosities. The swept volume ratio is unity for the two chambers and the phase lag of the volume variations between the cold and hot sides is 90°.
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